MENIIT
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JEE MAINS-2019

09-01-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Mathematics, Chemistry and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-MATHEMATICS

1. Let A $=\left\{\theta \in \left(\frac{-\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$. Then the sum of the element in A is

(1) π (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4*) $\frac{2\pi}{3}$

Sol. $z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$

$z = \frac{\left(3-4\sin^2\theta\right)+8i\sin\theta}{1+4\sin^2\theta}$

For purely imaginary real part should be zero.

i.e. $3 - 4\sin^2\theta = 0$.

i.e. $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$, Sum of all values is $-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3} = \frac{2\pi}{3}$

2. Let $f : R \to R$ be a function defined as $f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a+bx, & \text{if } 1 < x < 3 \\ b+5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$. Then f is

(1) continuous if a = – 5 and b = 10

(2) continuous if a = 5 and b = 5

(3) continuous if a = 0 and b = 5

(4*) not continuous for any values of a and b

Sol. For x = 1

R.H.L = a + b

L.H.L = 5

So to be continuous at x = 1

a + b = 5 …………(i)

for x = 3

R.H.L. = b + 15

L.H.L = a + 3b

b + 15 = a + 3b

a + 2b = 15 ………….(ii)

for x = 5

R.H.L = 30

L.H.L = b + 25

b + 25 = 30

b = 5.

From equation (ii)

a = 10

but a = 10 and b = 5 does not satisfied equation (i)

So f (x) is discontinuous for $a \in R$ and $b \in R$

3. The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point

(1) (– 3, 0, – 1) (2*) (3, 2, 1) (3) (3, 3, – 1) (4) (– 3, 1, 1)

Sol. Equation of required plane is

$(x + y + z - 1) + \lambda (2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda) x + (1 + 3\lambda) y + (1 - \lambda) = 0$

since given plane is parallel to y – axis $\Rightarrow 3\lambda + 1 = 0 \Rightarrow = -\frac{1}{3}$

Hence equation of plane is x + 4z – 7 = 0

4. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to

(1) 4 (2) 6 (3) 14 (4*) 8

Sol. $\frac{2^{403}}{15} = \frac{2^3.2^{400}}{15} = \frac{8.(1+15)^{100}}{15}$

$= \frac{8\left({}^{100}C_0 + {}^{100}C_1(15) + {}^{100}C_2(15)^2 + ...\right)}{15}$

$\frac{8}{15} + 8\left({}^{100}C_1(15) + {}^{100}C_2(15)^2 + ...\right)$

Remainder is 8.

5. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true?

(1) The lines are not concurrent.

(2) Each line passes through the origin.

(3) The lines are all parallel.

(4*) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

Sol. px + qy + r = 0

$px + qy + \left(\frac{-3p - 2q}{4}\right) = 0$

$$p\left(x-\frac{3}{4}\right)+q\left(y-\frac{2}{4}\right)=0$$

$$x=\frac{3}{4}\text{and } y=\frac{1}{2}.$$

6. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \text{ o } J \text{ o } f_1)(x) = f_3(x)$, then J(x) is equal to

(1) $f_1(x)$ (2) $f_2(x)$ (3*) $f_3(x)$ (4) $\frac{1}{x}f_3(x)$

Sol. $x \in R - \{0,1\}$

$$f_1(x)=\frac{1}{x}, f_2(x)=1-x, f_3(x)=\frac{1}{1-x}$$

Given $f_2(J(f_1(x))) = f_3(x)$

$$1-J(f_1(x))=f_3(x)$$

$$J(f_1(x))=1-f_3(x)=1-\frac{1}{1-x}$$

$$J(f_1(x))=\frac{x}{x-1}$$

$$J\left(\frac{1}{x}\right)=\frac{x}{x-1}=\frac{1}{1-\frac{1}{x}}$$

$$J(x)=\frac{1}{1-x}=f_3(x)$$

7. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is

(1) $2y\sqrt{3}=12x+1$ (2) $\sqrt{3}y=3x+1$ (3*) $\sqrt{3}\,y=x+3$ (4) $2\sqrt{3}\,y=-x-12$

Sol. $ty = x + t^2$

$$\left|\frac{3+t^2}{\sqrt{1+t^2}}\right|=3$$

$$\Rightarrow t=\sqrt{3}$$

$$\Rightarrow \sqrt{3}y=x+3$$

8. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is

(1*) 300 (2) 500 (3) 350 (4) 200

Sol. Number of ways = Total number of ways without restriction – When two specific boys are in team without any restriction, total number of ways of forming team is ${}^7C_3 \times {}^5C_2 = 350$ If two specific boys B_1, B_2 are in same team then total number of ways of forming team equals to ${}^5C_1 \times {}^5C_2 = 50$ ways total ways = 350 – 50 = 300 ways

9. If A $= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2*) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (3) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Sol. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

By using symmetry

$A^{-50} = \begin{bmatrix} \cos(-50\theta) & -\sin(-50\theta) \\ \sin(-50\theta) & \cos(-50\theta) \end{bmatrix}$

At $\theta = \frac{\pi}{12}$

$A^{-50} = \begin{bmatrix} \cos\frac{25\pi}{6} & \sin\frac{25\pi}{6} \\ -\sin\frac{25\pi}{6} & \cos\frac{25\pi}{6} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

10. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{2\sqrt{2}}$.

(2) exists and equals $\frac{1}{2\sqrt{2}\,(\sqrt{2}+1)}$.

(3) does not exist.

(4*) exists and equals $\frac{1}{4\sqrt{2}}$.

Sol. $(1 + x)^n \cong 1 + nx$ when $(x \to 0)$

So, $\sqrt{1+y^4} = 1+\frac{y^4}{2}$

$$\lim_{x\to\infty} \frac{\sqrt{2+\frac{y^4}{2}}-\sqrt{2}}{y^4}$$

$$= \frac{\sqrt{2}\left(1+\frac{y^4}{8}-1\right)}{y^4} = \frac{\sqrt{2}}{8} = \frac{1}{4\sqrt{2}}$$

11. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and = 4, then $\vec{a} \cdot \vec{c}$ is equal to

(1) 8 (2) $\frac{17}{2}$ (3) 9 (4*) $\frac{19}{2}$

Sol. $a = \hat{i} - \hat{j},\ b = \hat{i} + \hat{j} + \hat{k},\ c = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} \times \vec{c} + \vec{b} = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ x & y & z \end{vmatrix} + \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$\hat{i}(-z) - \hat{j}(z) + \hat{k}(y+x)$

$\Rightarrow 1 - z = 0 \Rightarrow z = 1,$

Also x + y = –1, and $\vec{a}.\vec{c} = 4 \Rightarrow x - y = 4$

$\Rightarrow x = \frac{3}{2},\ y = \frac{5}{2}$

$\therefore |\vec{c}|^2 = x^2 + y^2 + z^2 = \frac{9}{4} + \frac{25}{4} + 1 = \frac{38}{4} = \frac{19}{2}$

12. If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be

(1) 4 (2*) 2 (3) – 3 (4) – 2

Sol. $a + ar + ar^2 = xar$

since $a \neq 0$ so $\frac{r^2 + r + 1}{r} = x;\quad 1 + r + \frac{1}{r} = x$

$\because r + \frac{1}{r} \in (-\infty, -2] \cup [2, \infty) \Rightarrow \quad x \in (-\infty, -1] \cup [3, \infty)$

13. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then | tan θ | is equal to

(1) $\frac{4}{9}$ (2*) $\frac{8}{15}$ (3) $\frac{7}{17}$ (4) $\frac{8}{17}$

Sol. $y = x^2 + 2$ and $y = 10 + x^2$

Meet at $(\pm 2, 6)$

$\Rightarrow m_1 = 4$ and $m_2 = -4$

$|\tan\theta| = \frac{8}{15}$

14. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to

(1) – 512 (2) 256 (3) 512 (4*) – 256

Sol. $x^2 + 2x + 2 = 0 \Rightarrow (x + 1)^2 = -1$

$x = -1 \pm i = \sqrt{2}e^{i\left(\pm\frac{3\pi}{4}\right)}$

$\therefore \alpha^{15}, \beta^{15} = \left(\sqrt{20}\right)^{15} \times 2\cos\left(15.\frac{3\pi}{4}\right)$

$= 2^8\sqrt{2} \times \left(-\frac{1}{\sqrt{2}}\right) = -256$

15. The value of $\int_0^{\pi} |\cos x|^3\, dx$ is

(1*) $\frac{4}{3}$ (2) $\frac{2}{3}$ (3) $\frac{-4}{3}$ (4) 0

Sol. $\int_0^{\frac{\pi}{2}} \left(|\cos x|^3 + |\cos(\pi - x)|^3\right) dx$

$\Rightarrow 2\int_0^{\frac{\pi}{2}} |\cos x|^3\, dx$

$\Rightarrow 2\int_0^{\frac{\pi}{2}} (\cos x)^3\, dx$

$\Rightarrow 2\left(\frac{2}{3}\right) = \frac{4}{3}$ (By wallis formula)

16. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals

(1) $13 - 4\cos^2\theta + 6\cos^4\theta$

(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(4*) $13 - 4\cos^6\theta$

Sol. $3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6 \sin 2\theta + 4 \sin^6 \theta$

$= 9 + 3\sin^2 2\theta + 4 \sin^6 \theta$

$= 9 + 12\sin^2 \theta \cos^2 \theta + 4 (1 - \cos^2 \theta)^3$

$= 9 + 12(1 - \cos^2 \theta) \cos^2 \theta + 4(1 - 3\cos^2 \theta + 3\cos^4 \theta - \cos^6 \theta)$

$= 13 + 12\cos^2 \theta - 12\cos^4 \theta - 12\cos^2 \theta + 12\cos^4 \theta - 4\cos^6 \theta$

$= 13 - 4\cos^6 \theta$

17. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has a unique solution for $|a| = \sqrt{3}$.

(2) is inconsistent when a = 4.

(3*) is inconsistent when $|a| = \sqrt{3}$.

(4) has infinitely many solutions for a = 4.

Sol. By applying Crammer's Rule

$$d = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

$= 3(a^2 - 1) - 6 - 2(a^2 - 1) + 4$

$= a^2 - 1 - 2 = a^2 - 3$

If $|a| \neq \pm\sqrt{3} \Rightarrow$ system has unique solution

If $|a| = \sqrt{3}$ $\left.\begin{array}{r} x + y + z = 1 \\ 2x + 3y + 2z = 1 \\ 2x + 3y + 2z = \pm\sqrt{3} + 1 \end{array}\right\}$

Hence system is inconsistent for $|a| = \sqrt{3}$

18. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to

(1) $\frac{7}{64}$

(2*) $\frac{49}{16}$

(3) $\frac{13}{16}$

(4) $\frac{1}{4}$

Sol. $x\frac{dy}{dx} + 2y = x^2$

$\frac{dy}{dx} + \frac{2}{x}y = x$

This is linear differential equation in $\frac{dy}{dx}$

Integrating factor $= e^{\int \frac{2}{x}dx} = x^2$

Solution of differential equation is $yx^2 = \int x^3 dx$

$yx^2 = \frac{x^4}{4} + c$

Curve passes through (1, 1)

then $c = \frac{3}{4}$

$yx^2 = \frac{x^4 + 3}{4}$

Put $x = \frac{1}{2}$

$y\left(\frac{1}{4}\right) = \frac{\left(\frac{1}{2}\right)^2 + 3}{4}$

$y = \frac{49}{16}$

19. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is

(1) 18 (2) 22 (3) 16 (4*) 20

Sol. Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $\bar{x} = \frac{\sum x_i}{5} = 150 \Rightarrow \sum_{i=1}^{5} = 750$ ....(1)

$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} - (150)^2 = 18$

$\Rightarrow \sum x_i^2 = (22500 + 18) \times 5 \Rightarrow \sum_{i=1}^{5} x_i^2 = 112590$ ....(2)

Height of new student = 156 (Let x_6)

Then $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$\bar{x}_{new} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{906}{6} = 151$ ....(3)

Variance (new) $= \frac{\sum x_i^2}{6} - (\bar{x}_{new})^2$

from equation (2) and (3)

variance (new) $= \dfrac{112590 + (156)^2}{6} - (151)^2 = 22821 - 22801 = 20$

20. If the boolean expression (p ⊕ q) ∧ (~p Θ q) is equivalent to p ∧ q, where ⊕, Θ ∈ [∧, ∨], then the ordered pair (⊕,Θ) is:

(1) (∧, ∧) (2) (∨, ∨) (3) (∧, ∨) (4*) (∨, ∧)

Sol. Check all option repeatedly

(i) (A ∧ B) ∧ (~ A ∨ B) ≡ A ∧ (B ∧ (~ A ∨ B))

≡ A ∧ (B) ≡ A ∧ B

⇒ (i) is correct

(ii) (A ∧ B) ∧ (~ A ∧ B) ≡ (A ∧ ~ A) ∧ B

≡ f ∧ B ≡ f

(iii) (A ∨ B) ∧ (~ A ∨ B) ≡ B

(iv) (A ∨ B)(~ A ∨ B)

≡ B ∨ (A ∧ ~ A) = B ∨ f ≡ f

⇒ only (1) is correct

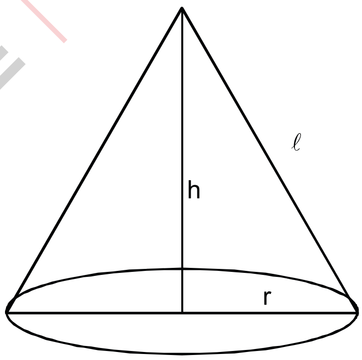
21. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is

(1) 6π (2) $\frac{4}{3}\pi$ (3*) $2\sqrt{3}\pi$ (4) $3\sqrt{3}\pi$

Sol. $\ell = 3$

$r^2 + h^2 = 9$

Volume of cone is $= \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}\pi h\left(9 - h^2\right)$

$\frac{dv}{dh} = \frac{1}{3}\pi\left(9 - 3h^2\right) = 0$

$9 - 3h^2 = 0$

$h^2 = 3$, $h = \sqrt{3}$

$V = \frac{1}{3}(\pi)(6)\sqrt{3} = 2\sqrt{3}\ \pi$

22. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals

(1) $\frac{49}{169}$ (2) $\frac{24}{169}$ (3) $\frac{52}{169}$ (4*) $\frac{25}{169}$

Sol. $P(x=1)=\frac{4}{52}\times\frac{48}{52}\times 2=\frac{24}{169}$

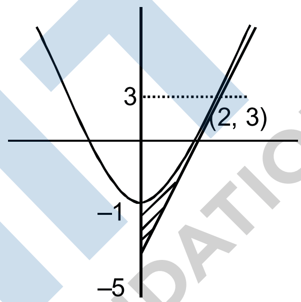
$P(x=2)=\frac{4}{52}\times\frac{4}{52}=\frac{1}{169}$

$\Rightarrow P(x=1)+P(x+2)=\frac{25}{169}$

23. The area (in s units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is:

(1) $\frac{14}{3}$ (2*) $\frac{8}{3}$ (3) $\frac{56}{3}$ (4) $\frac{32}{3}$

Sol. Area $=\int_{-5}^{3} x\,dy-\int_{-1}^{3} x\,dy$



$=\int_{-5}^{3}\left(\frac{y+5}{4}\right)-\int_{-1}^{3}\sqrt{y+1}\,dy$

$=\left|\frac{\frac{y^2}{2}+5y}{4}\right|_{-5}^{3}-\left|\frac{2}{3}(y+1)^{3/2}\right|_{-1}^{3}$

$=\left|\frac{\left(\frac{9}{2}+15\right)-\left(\frac{25}{2}+25\right)}{4}\right|=\left|\frac{16}{3}\right|=\frac{8}{3}$

24. Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S=\sum_{i=1}^{30} a_i$ and $T=\sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to

(1) 57 (2) 47 (3*) 52 (4) 42

Sol. $S=\sum_{i=1}^{30} a_i$, $T=\sum_{i=1}^{15} a_{2i-1}$, $a_5=27, S-2T=75$

Let $a_i = a + (i - 1)D$

$S = a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots + a_{30}$

$T = a_1 + a_3 + a_5 + \ldots\ldots\ldots\ldots + a_{29}$

$\therefore 2T = 2a_1 + 2a_3 + 2a_5 + \ldots\ldots\ldots\ldots + 2a_{29}$

$S - 2T = (a_2 - a_1) + (a_4 - a_3) + (a_6 - a_5) + \ldots\ldots\ldots\ldots + (a_{30} - a_{29}) = 75$

$= 15D$

But $S - 2T = 75 \Rightarrow 15D = 75 \Rightarrow D = 5$

Now $a_5 = 27 \Rightarrow a + 4D = 27$

$\therefore a = 27 - 20 \Rightarrow a = 7$

now $a_{10} = a + 9D$

$= 7 + 45 = 52$

25. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval:

(1) $(1, 3/2]$ (2) $(2, 3]$ (3) $(3, \infty)$ (4) $(3/2, 2]$

Sol. $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$

$\because$ e > 2 (given)

$e^2 > 4 \Rightarrow 1 + \frac{\sin^2\theta}{\cos^2\theta} > 4$

$\Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \tan^2\theta > 3$

$\because \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum $= 2\frac{\sin^2\theta}{\cos\theta} = 2\tan\theta\sin\theta$

$\because$ for $\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, $2\tan\theta\sin\theta$ is increasing function

Hence latus rectum $\in (3, \infty)$

26 Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then

(1) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P. (2*) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(3) a, b, c are in A.P. (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Sol. Length of direct common tangent for circle C_1 and C_2 is

$AB = \sqrt{(a+b)^2 - (a-b)^2}$

For C_2 and C_3

Length of direct common tangent is

$BC = \sqrt{(a+c)^2 - (a-c)^2}$

For C_1 and C_3

Length of direct common tangent is

$AC = \sqrt{(b+c)^2 - (b-c)^2}$

AB + BC = AC

$\sqrt{(a+b)^2-(a-b)^2}+\sqrt{(a+c)^2-(a-c)^2}=\sqrt{(b+c)^2-(b-c)^2}$

$\sqrt{ab}+\sqrt{ac}=\sqrt{bc}$

$\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}$

27. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$, then x is equal to

(1) $\frac{\sqrt{146}}{12}$ (2) $\frac{\sqrt{145}}{11}$ (3) $\frac{\sqrt{145}}{10}$ (4*) $\frac{\sqrt{145}}{12}$

Sol. $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x\geq\frac{3}{4}\right)$

$\cos^{-1}\left(\frac{2}{3x}\times\frac{3}{4x}-\sqrt{1-\frac{4}{9x^2}}\sqrt{1-\frac{9}{16x^2}}\right)=\frac{\pi}{2}$

$\Rightarrow \frac{1}{2x^2}=\frac{\sqrt{9x^2-4}\sqrt{16x^2-9}}{12x^2}$

$\Rightarrow 6=\sqrt{9x^2-4}\sqrt{16x^2-9}$

Square both side

$36 = 144x^4 - 81x^2 = 64x^2 + 36$

$\Rightarrow \quad 144x^4 = 145x^2$

$\Rightarrow \quad x^4=\frac{145x^2}{144} \quad \Rightarrow \quad x=\pm\frac{\sqrt{145}}{12},0$

$\therefore \quad x>\frac{3}{4} \quad \Rightarrow \quad x=\frac{\sqrt{145}}{12}$

28. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(1) $(5,2\sqrt{6})$ (2*) (8, 6) (3) $(6,4\sqrt{2})$ (4) (4, – 4)

Sol. Vertex is (2, 0)

a = 2

Any general point on given parabola can be taken as $(2 + 2t^2, 4t)$ $\forall t\in R$.

(8, 6) does not lie on this.

29. The equation of the line passing through (– 4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is

(1) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$ (2) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$

(3*) $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$ (4) $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

Sol. Let the line L be $\frac{x+4}{a}=\frac{y-3}{b}=\frac{z-1}{c}$

L || x + 2y – z – 5 = 0

L intersects $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$

$$\Rightarrow \begin{vmatrix} 3 & 0 & 1 \\ a & b & c \\ -3 & 2 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ 2a + 0b – 6c = 0

Also a + 2b – c = 0

$\therefore \frac{a}{3}=\frac{b}{-1}=\frac{c}{1}$

$\therefore$ L is $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$

30. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$ is equal to (where c is a constant of integration)

(1) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$ (2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(3) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ (4*) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

Sol. $\int x\sqrt{\frac{2\sin(x^2-1)(1-\cos(x^2-1))}{2\sin(x^2-1)(1+\cos(x^2-1))}}$

$$= \int x\frac{\sin\left(\frac{x^2-1}{2}\right)}{\cos\left(\frac{x^2-1}{2}\right)}dx$$

$$= \int x\tan\left(\frac{x^2-1}{2}\right)dx$$

Let $\frac{x^2-1}{2}=t \quad \Rightarrow \quad 2xdx = 2dt$

$= \int \tan(t)\,dt = \ell n\,|\sec t| + c$

$$= \ell n\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$$

PART-B-CHEMISTRY

31. The one that is extensively used as a piezoelectric material is:

(1*) Quartz (2) Amorphous silica (3) Mica (4) Tridymite

Sol. Materials those produce electric current when they are put under mechanical stress are called piezoelectric materials.

32. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

(1*) Both are stable (2) Li_2^+ is unstable and Li_2^- is stable

(3) Both are unstable (4) Li_2^+ is stable and Li_2^- is unstable

Sol. Both Li_2^+ and Li_2^- have same bond order. But the number of antibonding electrons is less in Li_2^+ than in Li_2^-

33. The isotopes of hydrogen are:

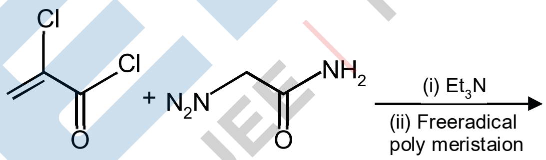
(1) Protium and deuterium only (2) Deuterium and tritium only

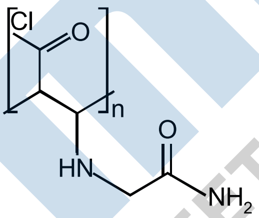
(3) Tritium and protium only (4*) Protium, deuterium and tritium

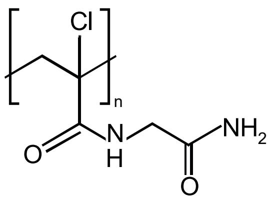
Sol. The isotopes are:

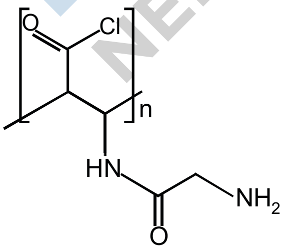
$^1_1H, ^2_1H$ and $^3_1H \equiv P, D, T$

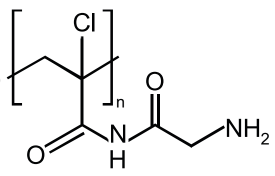
34. Major product of the following reaction is :

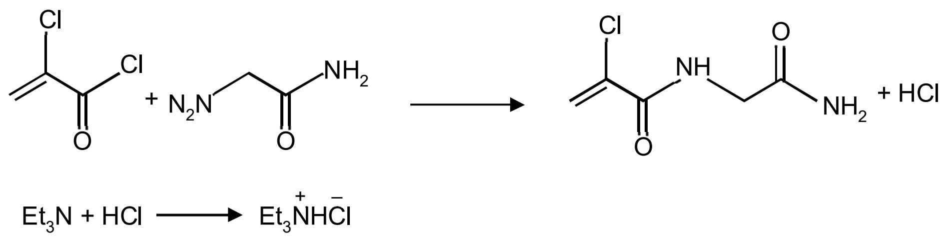


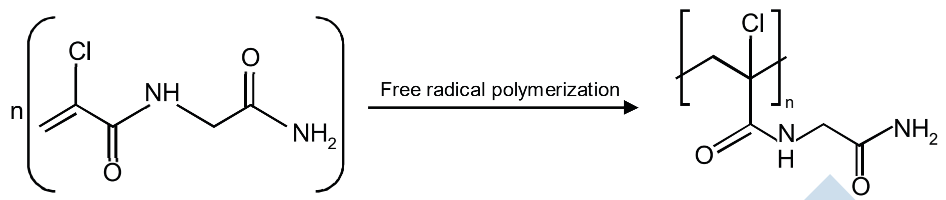
(1) 

(2*) 

(3) 

(4) 

Sol. 



35. In general, the properties that decrease and increase down a group in the periodic table, respectively are

(1) Electron gain enthalpy and electronegativity

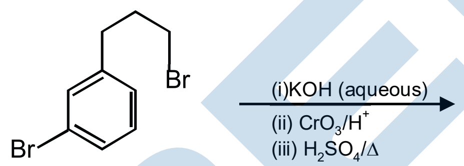
(2) Electronegativity and electron gain enthalpy

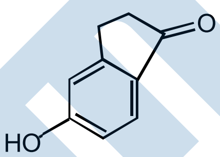
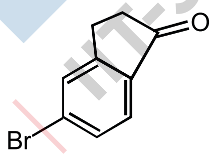
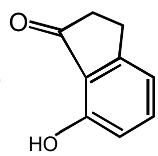
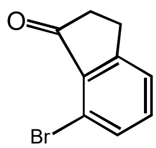
(3) Atomic radius and electronegativity

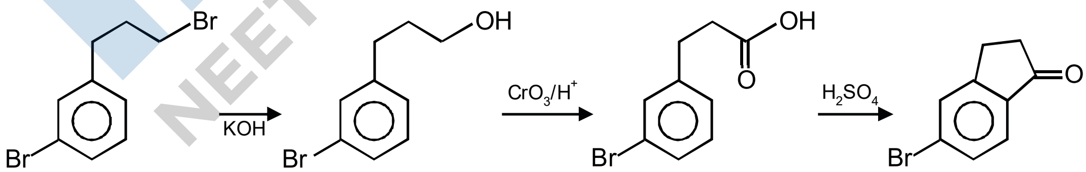
(4*) Electronegativity and atomic radius

Sol. On moving down a group, electronegativity decreases and atomic radius increases for representative elements.

36. The major product of the following reaction is:



(1)  (2)  (3)  (4) 

Sol. 

37. The correct match between Item-I and Item-II is:

	Item-I		Item-II
	(Drug)		(Test)
(A)	Chloroxylenol	(P)	Carbylamine test

(B) Norethindrone (Q) Sodium hydrogen-carbonate test

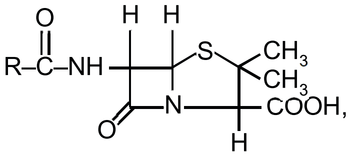
(C) Sulphapyridine (R) Ferric chloride test

(D) Penicillin (S) Bayer's test

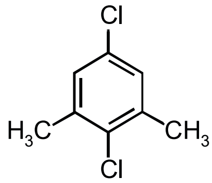
(1) A→R ; B→P ; C→S ; D→Q (2) A→Q ; B→P ; C→S ; D→R

(3) A→Q ; B→S ; C→P ; D→R (4*) A→R ; B→S ; C→P ; D→Q

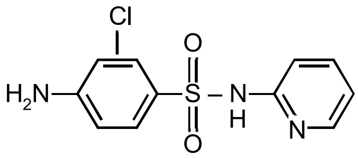
Sol.



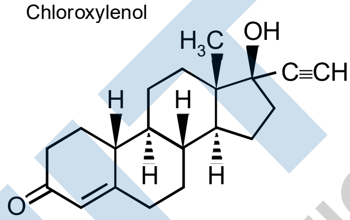
Penicillin



Chloroxylenol



Sulphapyridine



Norethindrone

38. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is:

(1) Both are paramagnetic with three unpaired electrons.

(2) Both absorb energies corresponding to their complementary colors.

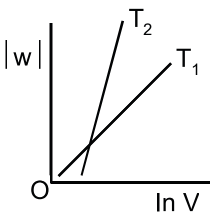
(3) Δ_0 value for (A) is less than that of (B).

(4*) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

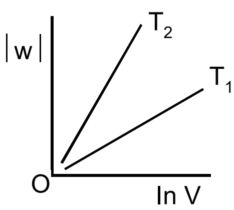
Sol. Δ_0 is calculated from the energies of absorbed radiation not from emitted radiation (complementary colour).

39. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

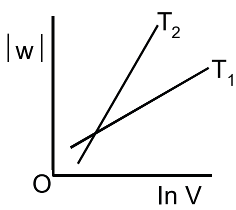
(1)



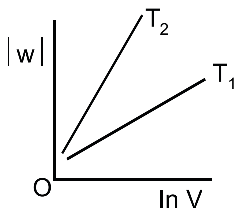
(2)



(3*)



(4)



Sol. Let the gas is expanded from V_1 to V at T_1 and from V_2 to V at T_2

$\therefore$ At T_1

$$|W_1| = nRT_1\ \ell n\frac{V}{V_1} = nRT(\ell nV - \ell nV_1)$$

Similarly at T_2

$|W_2| = nRT_2(\ell nV - \ell nV_2)$

$\therefore$ $W_1 = nRT_1\ \ell nV - nRT_1\ \ell nV_1$

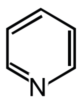
$W_2 = nRT_2\ell nV - nRT_2\ell nV_2$

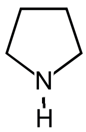
Slope of W_2 > Slope of W_1

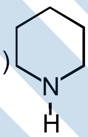
As $nRT_2 > nRT1(T_2 > T_1)$

$\therefore$ The intercept of W_2 is more negative than that of W_1 because $V_2 > V_1$.

40. Arrange the following amines in the decreasing order of basicity:

(I) 

(II) 

(III) 

(1) III > II > I (2) I > II > III (3*) III > I > II (4) I > III > II

Sol. In(III), nitrogen atom undergoes sp^3, in(I) sp^2 hybridization. In(II), the lone pair participate in resonance.

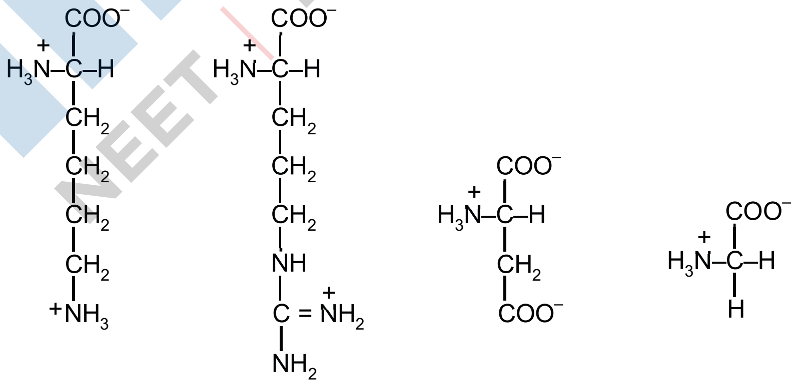
41. The increasing order of pKa of the following amino acids in aqueous solution is:

Gly Asp Lys Arg

(1) Asp < Gly < Arg < Lys (2) Arg < Lys < Gly < Asp

(3) Gly < Asp < Arg < Lys (4*) Asp < Gly < Lys < Arg

Sol.



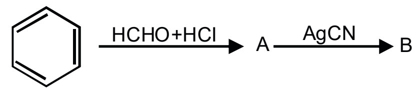
	* Lysine	* Arginine	Aspartate	Glycine
pH Value	**(9.8)**	**(10.8)**	**(3.0)**	**(6.0)**

42. The alkaline earth metal nitrate that does not crystallise with water molecules, is:

(1) $Ca(NO_3)_2$ (2) $Mg(NO_3)_2$ (3*) $Ba(NO_3)_2$ (4) $Sr(NO_3)_2$

Sol. Due to larger size of Ba^{2+} ion, $Ba(NO_3)_2$ cannot hold water molecules during crystallization.

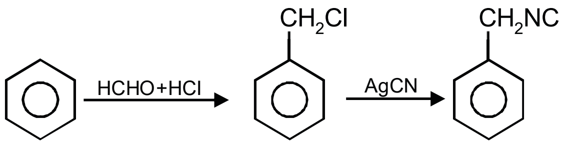
43. The compounds A and B in the following reaction are, respectively:



(1*) A = Benzyl chloride, B = Benzyl isocyanide (2) A = Benzyl alcohol, B = Benzyl cyanide

(3) A = Benzyl alcohol, B = Benzyl isocyanide (4) A = Benzyl chloride, B = Benzyl cyanide

Sol.



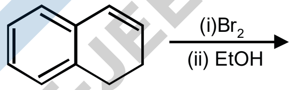
44. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is:

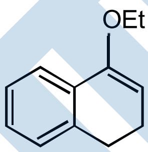
(1) 4.90 (2) 3.87 (3*) 5.92 (4) 6.93

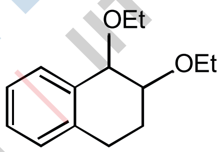
Sol. Maximum number of unpaired electrons of metal or metal ion in complexes = n = 5

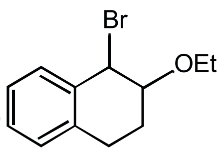
$\therefore \mu_s = \sqrt{n(n+2)} = \sqrt{35} = 5.916 \approx 5.92$

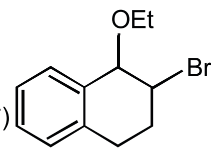
45. The major product of the following reaction is:



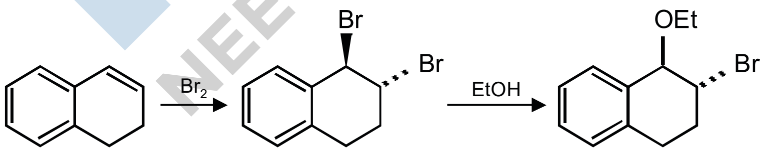
(1) 

(2) 

(3) 

(4*) 

Sol.



46. Which amongst the following is the strongest acid?

(1) CHI_3 (2*) $CH(CN)_3$ (3) $CHCl_3$ (4) $CHBr_3$

Sol. $CH(CN)_3 \rightleftharpoons \bar{C}(CN)_3 + H^+$

Negative charge of the conjugate base $\overset{-}{C}(CN)_3$ in extensively delocalized through the $C \equiv N$ group.

47. For emission line of atomic hydrogen from n_i = 8 to n_f =n, the plot of wave number ($\bar{\nu}$) againest $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)

(1*) Linear with slope – R_H (2) Non linear

(3) Linear with slope R_H (4) Linear with intercept – R_H

Sol. For emission line

$n_f < n_i$

$$\therefore \bar{\nu} = RZ^2\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] = R\left[\frac{1}{8^2} - \frac{1}{n^2}\right]$$

or, $\bar{\nu} = R_H\left(\frac{1}{64} - \frac{1}{n^2}\right)$

$$= \frac{R_H}{64} - \frac{R_H}{n^2}$$

$$\bar{\nu} = -R_H\left(\frac{1}{n^2}\right) + \frac{R_H}{64}$$

$\therefore$ y = mx + c

Slope = - R_H

48. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in JK^{-1} mol^{-1}, x is:

(1) $\frac{4+R}{2R}$ (2*) $\frac{4-R}{2R}$ (3) $\frac{2R}{4-R}$ (4) $\frac{2R}{4-R}$

Sol. PV = nRT

200 × 10 = (0.5 + x)R × 1000

On solving $x = \frac{4-R}{2R}$

49. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is: (Molar mass of $PbSO_4$ = 303 g mol^{-1})

(1) 15.2 (2) 22.8 (2) 11.4 (4) 7.6

Sol. $Pb(s) + SO_4^{2-} \longrightarrow PbSO_4 + 2e^-$

For 2F current passed, $PbSO_4$ deposited = 303 g/mol

For 0.05 F: $PbSO_4$ deposited $= \frac{0.05 \times 303}{2} = 7.6$ g

50. A water sample has ppm level concentration of the following metals:

Fe = 0.2; Mn = 5.0; Cu = 3.0, Zn = 5.0. The metal that makes the water sample unsuitable for drinking is:

(1) Cu (2*) Mn (3) Zn (4) Fe

Sol. The permissible level in ppm unit is

Fe = 0.2

Mn = 0.05

Cu = 3

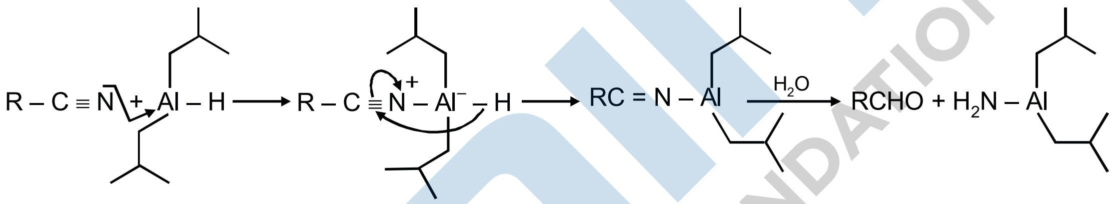
Zn = 5

Mn is higher

51. The major product of the following reaction is: $R-C\equiv N \xrightarrow[(2)H_2O]{(1)AlH(i\text{-}Bu)_2} ?$

(1) RCOOH (2) RCHO (3) RCH_2NH_2 (4) $RCONH_2$

Sol.



52. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is: [pk_b of NH_4OH = 4.7]

(1) 5.2 (2) 9.4 (3) 5.0 (4) 9.0

Sol. $\underset{\substack{2m.m \\ -}}{H_2SO_4} + \underset{\substack{6m.m \\ 2m.m}}{2NH_4OH} \longrightarrow \underset{2\,m.m}{(NH_4)_2SO_4} + 2H_2O$

$pOH = 4.7 + \log\frac{4}{2} = 5$

pH = 14 – 5 = 9

53. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:

(1) 4 (2) 12 (3) 8 (4) 16

Sol. Molality of $Na^+ = \left(\frac{w}{M}\times\frac{1000}{W}\right)\times 2$ (Na_2SO_4 contains two Na^+ ions)

$= \left[\left(\frac{92}{23}\times\frac{1000}{1000}\right)\right]\times 2 = 8$

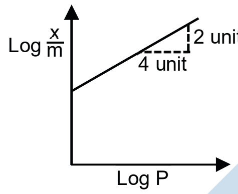
54. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and +3 oxidation states. This is due to:

(1) Diagonal relationship (2*) Inert pair effect

(3) Lanthanoid contraction (4) Lattice effect

Sol. The outermost electron configuration of Tl is $6s^2 6p^1$. The 6s electrons are strongly attracted towards the nucleus due to its more penetrating power and deshielding of d and f-electrons. Hence 6s electrons do not participate in bonding.

55. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p $\frac{x}{m}$. is proportional to:



(1) $p^{1/2}$ (2) $p^{1/4}$ (3) p (4) p^2

Sol. $\frac{x}{m} = KP^{1/n}$

or $\log\frac{x}{m} = \log K + \frac{1}{n}\log P$

Slope $= \frac{1}{n} = \frac{2}{4} = \frac{1}{2}$

$\therefore \frac{x}{m} \alpha P^{1/2}$

56. The ore that contains both iron and copper is:

(1) azurite (2) dolomite (3) malachite (4*) copper pyrites

Sol. Copper pyrites is $CuFeS_2$

Malachite: $CuCO_3.Cu(OH)_2$

Azurite : $2Cu(CO)_3.Cu(OH)_2$

Dolomite: $CaCO_3.MgCO_3$

57. Which one of the following statements regarding Henry's law is not correct?

(1*) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids

(2) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas

(3) Different gases have different K_H (Henry's law constant) values at the same temperature

(4) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

Sol. Gases having higher K_H value are less soluble.

58. The correct decreasing order for acid strength is:

(1) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(2) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

(3) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

(4*) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

Sol. Acid strength in this case varies directly with the electron withdrawing power of the groups attached to the α-carbon of CH3COOH. The order of electron withdrawing tendency is

$NO_2 > CN > F > Cl$

59. Correct statements among a to d regarding silicones are:

(a) They are polymers with hydrophobic character.

(b) They are biocompatible

(c) In general, they have high thermal stability and low dielectric strength.

(d) Usually, they are resistant to oxidation and used as greases.

(1) (a), (b), (c) and (d)
(2*) (a), (b) and (d) only
(3) (a), (b) and (c) only
(4) (a) and (b) only

Sol. Silicones are polymers and hydrophobic due to presence of alkyl groups. They are used as greases as some of them are cyclic.

60. The following results were obtained during kinetic studies of the reaction;

$$2A + B \longrightarrow \text{Product}$$

Experiment	[A] (in $molL^{-1}$)	[B] (in $molL^{-1}$)	Initial rate of reaction (in $molL^{-1}\,min^{-1}$)
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is:

(1) 1
(2) 100
(3*) 10
(4) 5

Sol. $R = k[A]^x\,[B]^y$

$6.93 \times 10^{-3} = k\,(0.1)^x\,(0.2)^y$ (i)

$6.93 \times 10^{-3} = k\,(0.1)^x\,(0.25)^y$ (ii)

$1.386 \times 10^{-2} = k\,(0.2)^x\,(0.3)^y$ (iii)

$\Rightarrow$ y = 0 (from (i) & (ii)), zero order w.r.t. B

x = 1 (from (i) & (iii))

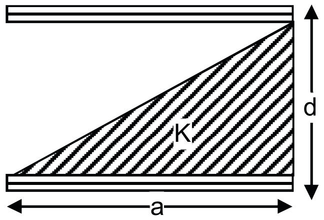
$\Rightarrow$ First order wrt A

$\Rightarrow 6.93 \times 10^{-3} = k\,(0.1)$

$\Rightarrow k = 6.93 \times 10^{-3}\ min^{-1}$

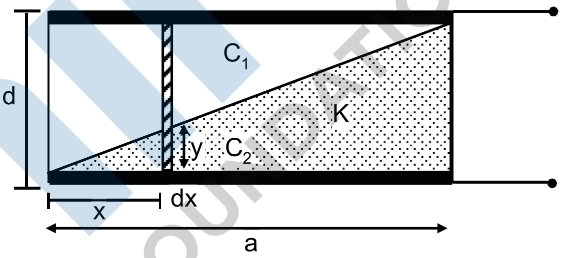
PART-C-PHYSICS

61. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d(d <<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is:



(1) $\frac{1}{2}\frac{K \in_0 a^2}{d}$ (2) $\frac{K \in_0 a^2}{2d(K+1)}$ (3) $\frac{K \in_0 a^2}{d}\ln K$ (4*) $\frac{K \in_0 a^2}{d(K-1)}\ln K$

Sol. Let's consider a strip of thickness dx at a distance of x from the left end as shown in the figure.



$$\frac{y}{x} = \frac{d}{a}$$

$$\Rightarrow \quad y = \left(\frac{d}{a}\right)x$$

$$C_1 = \frac{\varepsilon_0 adx}{(d-y)} \quad ; \quad C_2 = \frac{k\varepsilon_0 adx}{y}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{k\varepsilon_0 adx}{kd+(1-k)y}$$

Now integrating it from 0 to a

$$\int_0^a \frac{k\varepsilon_0 adx}{kd+(1-k)\frac{d}{a}x} = \frac{k\varepsilon_0 a^2 \ell nk}{d(k-1)}$$

62. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is :

(1*) 1.0% (2) 0.5% (3) 2.0 % (4) 2.5%

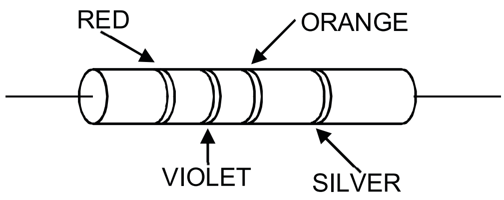
Sol. $R = \frac{\rho\ell}{A}$

$$R = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V}$$

$$\frac{dR}{R} = \frac{2d\ell}{\ell}$$

Hence, $\frac{dR}{R} = 1\%$.

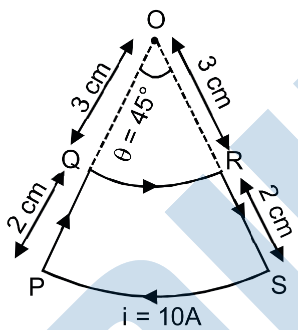
63. A resistance is shown in the figure. Its value and tolerance are given respectively by:



(1) 270 Ω, 10% (2) 27 kΩ, 20% (3) 270 Ω, 5% (4*) 27 kΩ, 10%

Sol. Fact based.

64. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to:



(1) 1.5×10^{-5} T (2*) 1.0×10^{-5} T (3*) 1.0×10^{-7} T (4) 1.5×10^{-7} T

Sol. Magnetic field at centre of an area subtending angle θ at the centre $\frac{\mu_0 I}{4\pi r}\theta$.

$$B = \left(\frac{\mu_0}{4\pi}\right) \times 10\left(\frac{1}{3\times10^{-2}} - \frac{1}{5\times10^{-2}}\right)\frac{\pi}{4}$$

$$= B = \frac{\pi}{30} \times 10^{-4} = \frac{\pi}{3} \times 10^{-5} \approx 10^{-5}$$

65. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively:

(1) 20 days and 10 days (2) 5 days and 10 days

(3*) 20 days and 5 days (4) 10 days and 40 days

Sol. Activity = λ (number of atoms)

$10 = \lambda_A N_A$ ...(1)

$20 = \lambda_B N_B$ ...(2)

$N_A = 2N_B$ ...(3)

Solving we get, $\frac{\lambda_A}{\lambda_B} = \frac{1}{4}$

66. A conducting circular loop made of a thin wire, has area 3.5×10^{-3} m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field B(t) = (0.4T)sin(50πt). The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to :

(1) 14 mC (2) 6 mC (3) 21 mC (4) 7 mC

Ans. 140mC (**Bonus**)

Sol. $B(t) = 0.4 \text{ Sin } (50\pi \times 10^{-2})$

$$= 0.4\text{Sin}\left(\frac{50\pi}{100}\right) = 0.4$$

$$\Delta q = \frac{-\Delta Q}{R} = \frac{\Delta Q}{R}$$

$$= \frac{0.4 \times 3.5 \times 10^{-3}}{10} = 140 \text{ mC}$$

No option matching.

67. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is:

(1) 1.1 cm away from the lens (2) 0

(3) 0.55 cm towards the lens (4*) 0.55 cm away from the lens

Sol. If u = –10 cm

v = +10 cm

$\Rightarrow$ f = 5 cm

Glass plate shift $= t\left(1 - \frac{1}{\mu}\right) = 1.5\left(1 - \frac{2}{3}\right) = 0.5$ cm

So, new u = 10 – 0.5 = 9.5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$$

After solving we get,

$v = \frac{47.5}{4.5}$. Hence, shift $\frac{47.5}{4.5} - 10 = \left(\frac{2.5}{4.5}\right) = 0.55$ cm

68. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is:

(1) $\frac{2L}{m}$ (2*) $\frac{L}{2m}$ (3) $\frac{L}{m}$ (4) $\frac{4L}{m}$

Sol. Based on Kepler's law.

$$\frac{dA}{dt} = \frac{L}{2m}$$

69. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be:

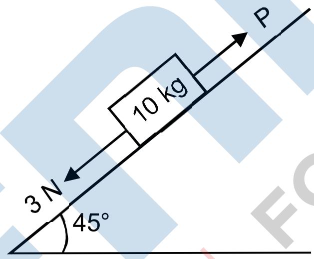
(1) $18.9 \times 10^{-8}\hat{k}T$ (2) $6.3 \times 10^{-8}\hat{k}T$ (3*) $2.1 \times 10^{-8}\hat{k}T$ (4) $18.9 \times 10^{8}\ \hat{k}\ T$

Sol. $\frac{E}{B} = C$

$$B = \frac{E}{C} = \frac{6.3 \times 10^{27}}{3 \times 10^{8}} = 2.1 \times 10^{19}$$

70. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward?

(take g = 10 ms^{-2})



(1) 23 N (2) 25 N (3) 18 N (4*) 32 N

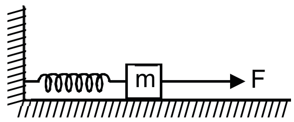
Sol. For equilibrium of the block net force should be zero. Hence we can write.

mg sin θ + 3 = P + friction

mg sin θ + 3 = P + μmg cos θ.

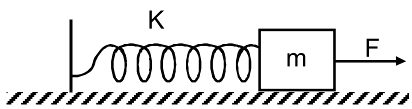
After solving, we get, P = 32 N.

71. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:



(1) $\frac{2F}{\sqrt{mk}}$ (2) $\frac{\pi F}{\sqrt{mk}}$ (3*) $\frac{F}{\sqrt{mk}}$ (4) $\frac{F}{\pi\sqrt{mk}}$

Sol. When $V_{max} \Rightarrow$ acceleration = 0



$\Rightarrow \quad x = \frac{F}{K}$

Apply work energy theorem

$W_{sp} + W_F = \Delta K.E.$

$-\frac{1}{2}Kx^2 + F.x = \Delta K.E. \quad ; \quad -\frac{1}{2}K\frac{F^2}{K^2} + \frac{F^2}{K} = \frac{1}{2}mu_{max}^2$

$\frac{F^2}{2K} = \frac{1}{2}mu_{max}^2 \quad ; \quad \frac{F}{\sqrt{mK}} = V_{max}$

72. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is:

(1) R (2*) $\frac{R}{\sqrt{2}}$ (3) $R\sqrt{2}$ (4) $\frac{R}{\sqrt{5}}$

Sol. Electric field

$E = \frac{KQx}{\left(x^2 + R^2\right)^{3/2}}$

For maxima $\frac{dE}{dx} = 0$

After solving we get, $\left(x \pm \frac{R}{\sqrt{2}}\right)$

73. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$, is close to:

(1) 0.45 (2*) 3.16 (3) 0.32 (4) 2.24

Sol. $\frac{(V_{RMS})_{He}}{(V_{RMS})_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}}$

$= \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

74. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 m^2 /(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to :

(1) 2 Ωm (2) 0.2 Ωm (3*) 0.4 Ωm (4) 4 Ωm

Sol. Use $I = neAv_d$ and $\mu = \frac{v_d}{E}$

75. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9×10^{28} /m^3 the value of v in mm/s is close to :

(Take charge of electron to be = 1.6×10^{-19} C)

(1) 3 (2) 2 (3*) 0.02 (4) 0.2

Sol. $i = ne AV_d$

$1.5 = 9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times V_d$

Vd = 0.02

76. Surface of certain metal is first illuminated with light of wavelength λ_1 = 350 nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to:

(Energy of photon = $\frac{1240}{\lambda(\text{in nm})}$eV)

(1) 1.4 (2) 5.6 (3) 2.5 (4*) 1.8

Sol. $\frac{1240}{350} - \phi = (KE)_I = 4x$ ...(1)

$\frac{1240}{540} - \phi = (KE)_{II} = x$ ...(2)

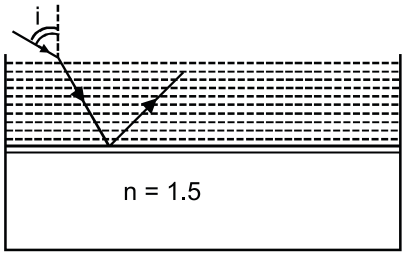
(1) – (2)

$\frac{1240}{340} - \frac{1240}{540} = 3.542 - 2.296 = 3x$

1.246 = 3x ; x = 0.41

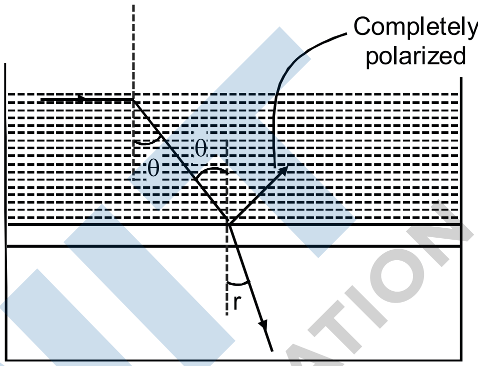
ϕ = 2.296 – 0.41 = 1.886

77. Consider a tank made of glass(refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is:



(1*) $\frac{3}{\sqrt{5}}$ (2) $\frac{4}{3}$ (3) $\frac{5}{\sqrt{3}}$ (4) $\sqrt{\frac{5}{3}}$

Sol. Sin 90° = μ sin θ



$\Rightarrow \quad \sin\theta = \frac{1}{\mu}$

μ sin θ = 1.5 sin r

μ tan θ = 1.5

$\Rightarrow \quad \tan\theta = \frac{1.5}{\mu}$

$\sin\theta = \frac{3}{\sqrt{9+4\mu^2}} = \frac{1}{\mu}$

$9\mu^2 = 9 + 4\mu^2$

$\Rightarrow \quad \mu = \frac{3}{\sqrt{5}}$

78. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:

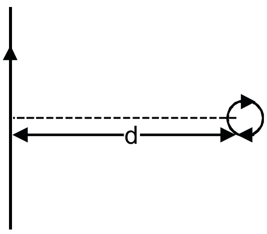
(1*) 2600 A/m (2) 285 A/m (3) 1200 A/m (4) 520 A/m

Sol. $B = \mu_o H$; $\mu_o ni = \mu_o H$

$\frac{100}{0.2} \times 5.2 = H$

H = 2600 A/m

79. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d >> a). If the loop applies a force F on the wire then



(1*) $F \propto \left(\frac{a}{d}\right)^2$ (2) $F \propto \left(\frac{a}{d}\right)$ (3) $F \propto \left(\frac{a^2}{d^3}\right)$ (4) F = 0

Sol. Force on one pole

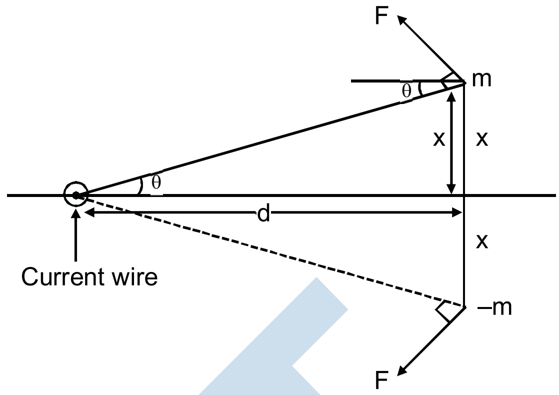
$$F = \frac{m\mu_0 I}{2\pi\sqrt{d^2 + x^2}}$$ m = pole

Strength

Total force = 2F sin θ

$$= \frac{2 \times \mu_0 Im \times x}{2\pi\sqrt{d^2+a^2}\sqrt{d^2+a^2}} = \frac{\mu_0 Imx}{\pi|d^2+a^2|}$$

$$= m^2x = M = I\,\pi a^2$$



Total force $= \frac{\mu_0 Ia^2}{2(d^2+a^2)}$

$$\approx \frac{\mu_0 Ia^2}{2d^2} \quad [\because d >> a]$$

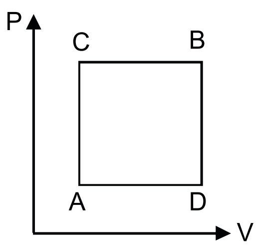
80. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

(1) $\frac{F}{2A\alpha\Delta T}$ (2*) $\frac{F}{A\alpha\Delta T}$ (3) $\frac{F}{A\alpha(\Delta T - 273)}$ (4) $\frac{2F}{A\alpha\Delta T}$

Sol. $\Delta L = L \propto \Delta T$

Strain $= \frac{\Delta L}{L} = \alpha\Delta T$; $Y = \frac{F}{A \propto \Delta T}$

81. A gas can be taken from A to B via two different processes ACB and ADB.



When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat flow into the system in path ADB is:

(1) 100 J (2*) 40 J (3) 80 J (4) 20 J

Sol. As temperature at point A and C is same.

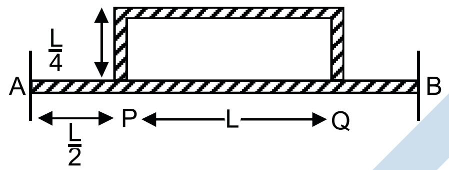
$\therefore$ Internal energy change will be same.

Q – W = Q' – W'

60 – 30 = Q' – 10

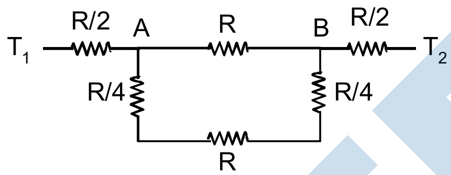
Q' = 40 J

82. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to:



(1) 60 °C (2*) 45 °C (3) 35 °C (4) 75 °C

Sol. $T_A - T_B = \frac{T_1 - T_2}{\frac{8R}{5}} \times \frac{3R}{5} = \frac{3}{8} \times 120 = 45$



83. Three charges +Q, q, +Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by +Q, placed at x = 0, is zero, then value of q is:

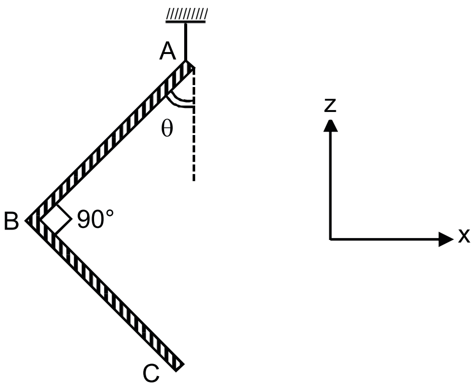
(1) + Q/2 (2*) – Q/4 (3) – Q/2 (4) + Q/4

Sol. $\frac{k\theta q}{\left(\frac{d}{2}\right)^2} = \frac{k\theta^2}{\left(\frac{3d}{2}\right)^2}$

$\Rightarrow \quad 4q = \frac{4Q}{9}$

$q = \frac{Q}{9}$

84. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then:



(1) $\tan\theta = \frac{1}{2\sqrt{3}}$ (2*) $\tan\theta = \frac{1}{3}$ (3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = \frac{2}{\sqrt{3}}$

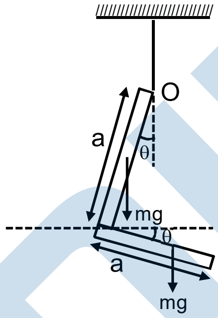
Sol. Lets considered mass of each rod is m for stable equilibrium the torque about point O should be zero.

Torque balance about O

$$mg\frac{a}{2}\sin\theta = mg\left(\frac{a}{2}\cos\theta - a\sin\theta\right)$$

$$\tan\theta = \frac{1}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{1}{3}\right)$$



85. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 16 : 9 (2) 4 : 1 (3) 5 : 3 (4*) 25 : 9

Sol. $\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right) = \frac{16}{1}$; $\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{4}{1}$

$$\Rightarrow \quad 3\sqrt{I_1} = 5\sqrt{I_2}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{25}{9}$$

86. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m << M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to:

(1) $\frac{g}{30}$ (2) $\frac{g}{20}$ (3*) $\frac{g}{5}$ (4) $\frac{g}{10}$

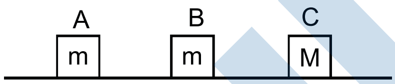
Sol. $v = \sqrt{T/\mu} = \sqrt{M\frac{g}{\mu}}$

$$\frac{\sqrt{g^2 + a^2}}{g} = \left(\frac{60.5}{60}\right)^2$$

$1 + \frac{1}{2}\frac{a^2}{g^2} = 1 + \frac{1}{60}$ using by binomial approximation.

$\Rightarrow \quad a = \frac{g}{\sqrt{30}}$

87. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of M/m?



(1) 5 (2) 2 (3) 3 (4*) 4

Sol. Apply LMC (Linear Momentum Conservation)

mv = (2m + M)v'

$v' = \frac{mv}{2m + M}$

Initial energy

$\frac{1}{2}mv^2$

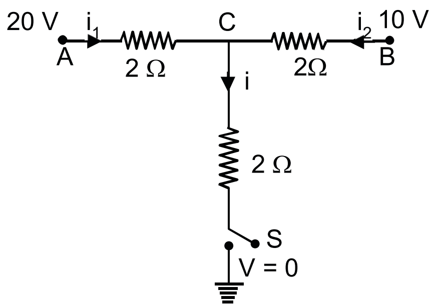
Final energy

$\frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$

Initial kinetic energy – Final kinetic energy $= \frac{5}{6}$ of initial kinetic energy.

After solving, we get, $\frac{M}{m} = 4$.

88. When the switch S, in the circuit shown, is closed, then the value of current i will be:



(1*) 5 A (2) 3 A (3) 2 A (4) 4 A

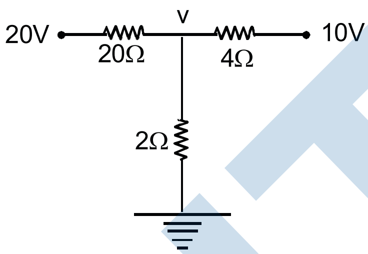
Sol. $i_3 + i_2 = i_1$

$\frac{20 - v}{2} + \frac{10 - v}{4} = \frac{v}{2}$

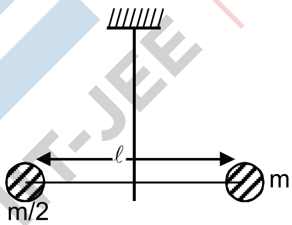
v = 10 V

$\Rightarrow \quad i_1 = \frac{10}{2}$

= 5 amp.



89. Two masses m $\frac{m}{2}$ and are connected at the two ends of a massless rigid rod of length ℓ. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:



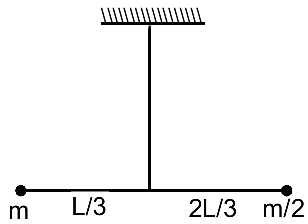
(1) $\frac{k\theta_0^2}{2\ell}$ (2*) $\frac{k\theta_0^2}{\ell}$ (3) $\frac{2k\theta_0^2}{\ell}$ (4) $\frac{3k\theta_0^2}{\ell}$

Sol. $\Omega = \sqrt{\frac{k}{I}}$; $\omega = \theta_0 \times \Omega$

$T = m\omega^2 \frac{\ell}{3}$

$T = m\omega^2 \frac{\ell}{3}\theta_0 \frac{k}{I}$ where $I = m\frac{\ell^2}{3}$

$= \frac{\theta_0^2 k}{\ell}$



90. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

(1) $y^2 = x + \text{constant}$

(2*) $y^2 = x^2 + \text{constant}$

(3) $xy = \text{constant}$

(4) $y = x^2 + \text{constant}$

Sol. $\frac{dx}{dt} = y$; $\frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x}$

$\Rightarrow \quad y^2 = x^2 + c$